Professur für Kontinuumsmechanik

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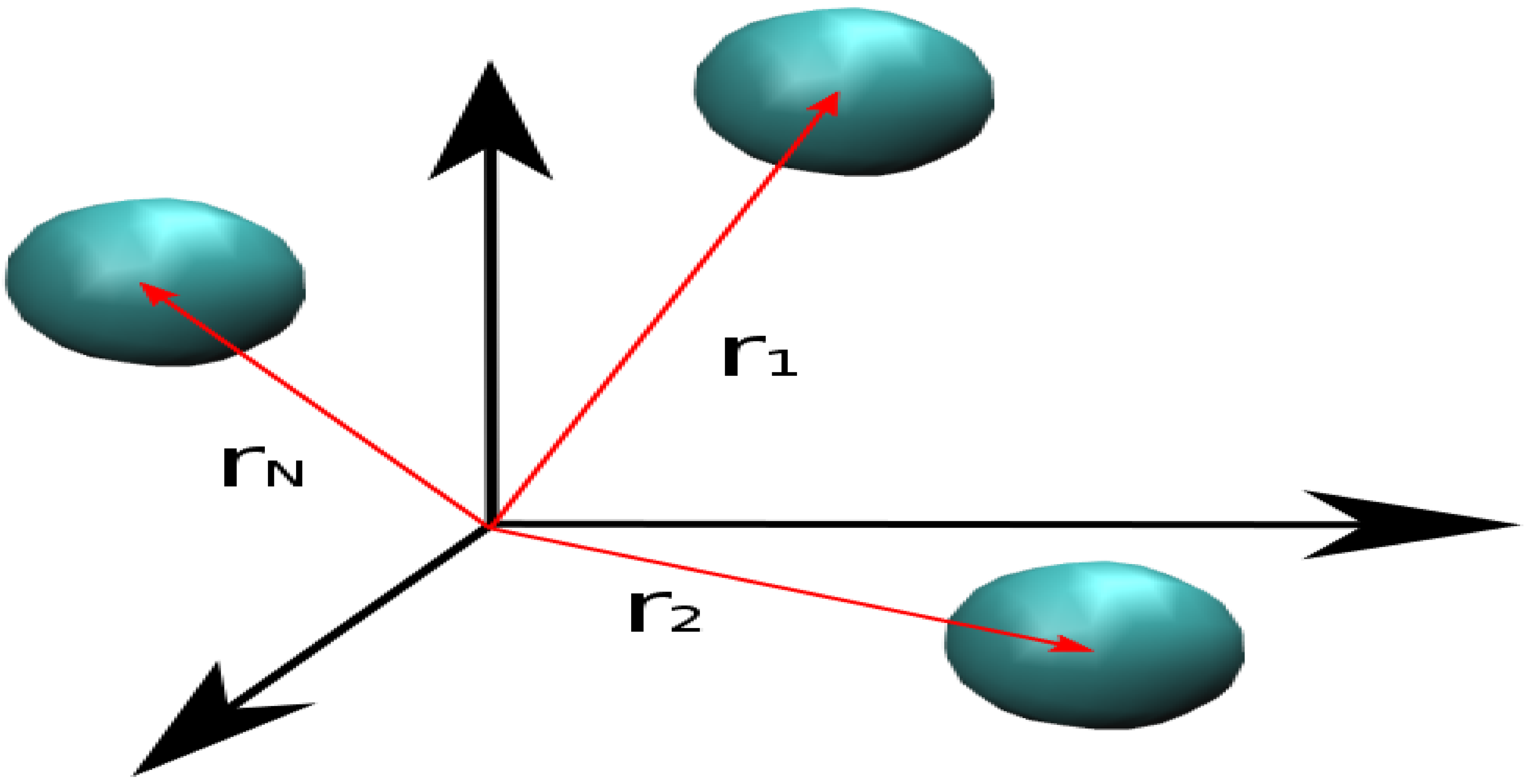
Do you want to win 200 Euros?*



Part A (150 Euros)

Let $\mathbf{R} = (\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$ represent the coordinates $\mathbf{r}_i \in \mathbb{R}^3, i = 1, \dots, N$ of N identical particles in three dimensions.

Can you find a representation/map $\mathcal{X}(\mathbf{R}) = \mathcal{X}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$ with the following properties:

- $\mathcal{X}(\mathbf{R})$ is invariant to permutations of the particles i.e.:

$$\mathcal{X}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = \mathcal{X}(\mathbf{r}_{\sigma(1)}, \mathbf{r}_{\sigma(2)}, \dots, \mathbf{r}_{\sigma(N)})$$

  where $(\sigma(1), \sigma(2), \dots, \sigma(N))$ is is any permutation of the indices $1, 2, \dots, N$.

- $\mathcal{X}(\mathbf{R})$ is invariant to rigid-body motions of the particles i.e.:

$$\mathcal{X}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = \mathcal{X}(\mathbf{r}_1 + \mathbf{c}, \mathbf{r}_2 + \mathbf{c}, \dots, \mathbf{r}_N + \mathbf{c}), \quad \forall \mathbf{c} \in \mathbb{R}^3$$

  and:

$$\mathcal{X}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = \mathcal{X}(\mathbf{Q}\mathbf{r}_1, \mathbf{Q}\mathbf{r}_2, \dots, \mathbf{Q}\mathbf{r}_N)$$

  for any orthogonal tensor $\mathbf{Q}$ in $\mathbb{R}^3$.

- If $\mathcal{X}(\mathbf{R}_1) = \mathcal{X}(\mathbf{R}_2)$ for two configurations $\mathbf{R}_1, \mathbf{R}_2$ of the N particles, then $\mathbf{R}_1$, $\mathbf{R}_2$ correspond to the same configuration, up to permutation of particles and rigid-body motion.

Part B (50 Euros if at least one of the following are fullfilled in addition to requirements in Part A)

- $\mathcal{X}(\mathbf{R})$ is "easy" to invert i.e. one can easily find a configuration $\mathbf{R}$ of particles (up to permutations and rigid-body motion) given the value of $\mathcal{X}$.
- $\mathcal{X}$ is decomposable into one-, two-, three-, etc. particle terms, e.g.:

$$\mathcal{X}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = \sum_{i_1=1}^{N} X_1(\mathbf{r}_{i_1}) + \sum_{i_1=1}^{N}\sum_{i_2=1}^{N} X_2(\mathbf{r}_{i_1}, \mathbf{r}_{i_2}) + \sum_{i_1=1}^{N}\sum_{i_2=1}^{N}\sum_{i_3=1}^{N} X_3(\mathbf{r}_{i_1}, \mathbf{r}_{i_2}, \mathbf{r}_{i_3}) + \dots$$

*Rules

- Answers must be submitted electronically, in PDF, to p.s.koutsourelakis@tum.de
- Only the first person to correctly respond will receive the prize
- Submissions must provide a correct answer to Part A to be eligible for the prize.
- Only students (Bachelors or M.Sc) are eligible to receive the prize
- For further information, please contact Prof. Koutsourelakis by email at p.s.koutsourelakis@tum.de